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Simulations of gear-based systems including backlash

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Abstract

The goal of this thesis is to develop a simulation system for geartrain models. This system incorporates the nonlinear behavior caused by backlash and will model this behavior using an impact pair. The simulations were used to analyze the behavior of controlled systems where the output state cannot be measured. This set of criteria allows for the application of the simulation as a tool to model the behavior of a control valve in the Lehigh University Engine Testing Laboratory dynamometer system. Using this model in Simulink, simulations were made to verify the system components and to evaluate controllers' responses. Different controllers were evaluated and the merits of a feedforward loop to compensate for the clearance in the gears were scrutinized. The system was also be modified to describe a system with a translational output. This system was developed independently by an undergraduate team and the combination of the simulation and the physical apparatus will be used in an instructional setting to explore system modeling and measurement. The general simulation was proven to be robust and to accurately model the behavior of a geartrain with backlash. All of the controllers tested were shown to be capable of controlling the output of the system. The feedforward loop was shown to reduce the overall output error in the system, but at the expense of many of the benefits of the proportional and integral controllers. The ability to shift the set point of a system with backlash has been shown to be highly beneficial particularly in systems with high levels of damping.

Introduction

This inquiry was undertaken in the pursuit of an effective control mechanism for a system with a nonlinear response, particularly geartrain backlash, and where the output state is not measurable. There was a requirement that a method be developed to compare the abilities of different controllers in such a situation. A model was developed to replicate the dynamics of a simple geartrain. Combined with models for a motor and a load, simulations could be performed where different controllers would be tested under identical conditions and compared with one another.

There are many systems where there are no measurements of the output state available. As measurements are required to drive closed loop controllers, these systems are particularly difficult to control effectively. One example of interest is that of a geartrain system where only the input side of the gear pair can be monitored. As it is the output of such a system which is important to control, this situation can produce complications in the development of adequate controllers. These types of situations arise in many mechanical systems and would be of critical importance in positioning systems and other high precision mechanisms.

The physical system upon which this project was based is a flow control valve used on a Lehigh University engine laboratory dynamometer, as shown in Figure 1. The valve is used to control the flow of coolant to the engine being tested in order to maintain a desired temperature in the engine. The total temperature range in the engine may be as small as ten degrees Fahrenheit and the controller is expected to keep the engine temperature within that ten degree band. In order to accomplish this level of control in such a high flow system, a number of actions were implemented. Fluctuations in source water temperature and pressure have been minimized by the use of a large holding tank

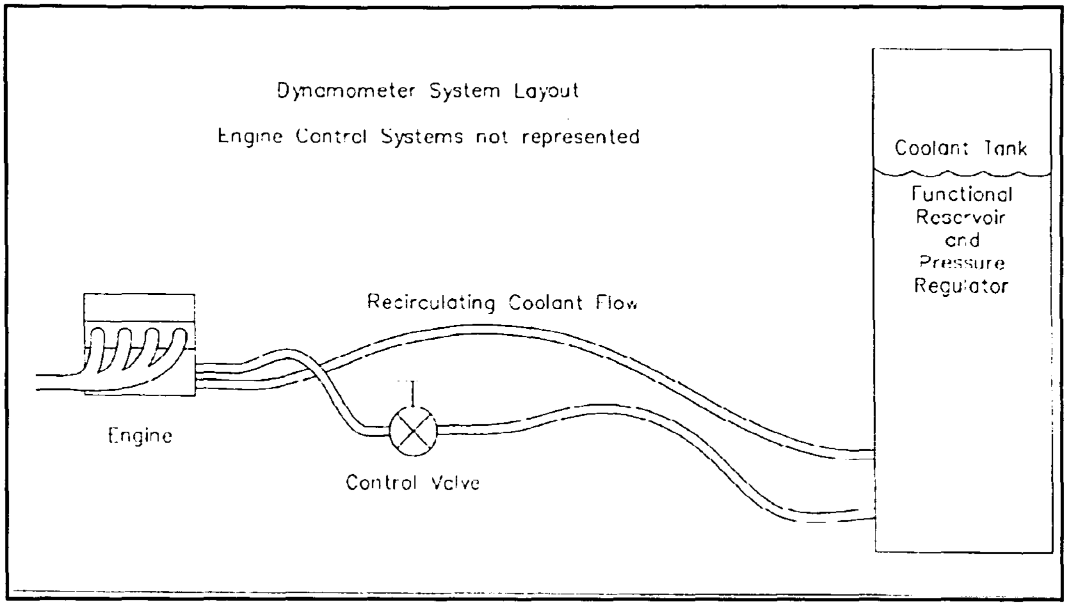


Figure 1: Engine Laboratory Dynamometer System Layout

which has a controlled volume of water. The size of the tank has been considered to prevent coolant recirculation from having more than a minimal transient response to the thermal capacity of the system. This system has alleviated early problems with temperature variations due to the accumulation of high temperature fluid during testing as well as variations in coolant flow due to source pressure variations. This configuration would provide a consistent flow of reasonably consistent temperature coolant to the engine. Additional testing showed that the system needed to be further controlled in order to maintain an engine temperature during testing. This control was to be accomplished with a computer controlled valve, using a feedback system based on the engine temperature. In practice, it was discovered that the valve system exhibited a nonlinear response to inputs that was symptomatically similar to backlash. Backlash manifests as a discontinuity in an output signal whenever direction is changed. Backlash is also characterized by the additional impacts when contact between components is resumed.

The primary objective of this undertaking is to evaluate different control methods applied to a nonlinear system with symptomatic backlash. This will be accomplished by creating a model based on accepted dynamic models which can be used to test the controllers. Simulink will be used as an industry standard and robust programming environment with generally high versatility. Once a dynamic model of a geartrain system is developed it will be tested with various standard inputs in order to validate its accuracy as an effective model and create a baseline response against which to compare the controlled system. Finally, different controllers will be tested and compared.

A secondary objective of this undertaking is to create effective dynamic models which may be used in an instructional setting to demonstrate nonlinear dynamic behavior to engineering students both from a modeling/simulation point of view and from a measurement perspective. This intent weighed heavily in the selection of Simulink as the modeling environment and also helped to drive the model format. The parameters most likely to be changed in the application of the model to different systems are thus required to be easily accessible and well documented. This should allow for the eventual inclusion of this model into other system models with minimal functional changes.

While the stated primary objective will drive the process of the inquiry, the practical design of the simulations and models will be driven to a large extent by the secondary objective of creating an instructional tool. The characteristics that will make the models useful in the classroom will also contribute to the future versatility of the system model as an analytical tool. The primary characteristic that will enable the models to be useful in other analyses is the design intent of the model to be modified. These changes could reflect the characteristics of a completely different physical system as required by different users. Another characteristic that will allow for simple modification of the system to reflect various systems is the modular construction of the component models. This would allow for the model developed for a single gear pair system to be expanded to model a two geartrain system with two gear pair interactions. This aspect was demonstrated using the controller determined to be the most effective at controlling nonlinear systems.

The valve system dynamics are very similar to those of geartrain backlash. A globe valve uses rotational motion to cause a translation in a plug which impedes the flow of a fluid, as seen in Figure 2. There is a disconnect between the rotation of the input shaft and changes in the valve position, this manifests when the direction of rotation of the shaft changes. The primary difference in the dynamics of the valve system compared to a geartrain is the magnitude of the clearance. Where in a gear the clearance might be 2 degrees of rotation, the valve system might have 20 degrees of rotation clearance when rotational direction is changed. Such a large clearance would increase the effects of the impact due to the increased speed and momentum of the driven components. However, as dynamic models, the systems are comparable and a geartrain could be substituted for the valve model in a simulation environment.

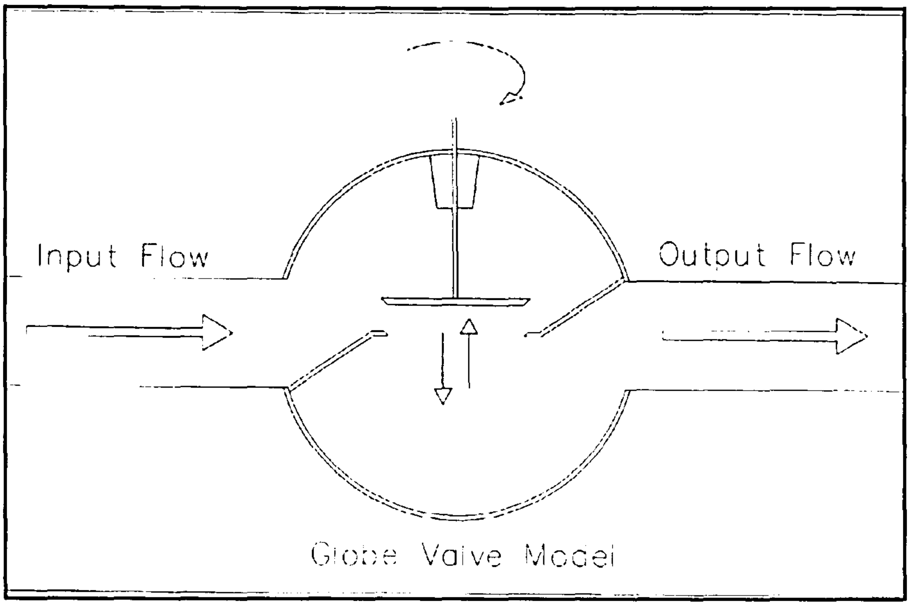


Figure 2: Valve Layout View

Background

There have been many impediments to the evaluation of controllers being applied to simple nonlinear systems. One primary impediment has been the lack of need for high precision control of highly nonlinear systems. Standard controllers can perform well enough for rough control of most nonlinear systems. As expectations of controller ability increase and they are applied to a greater variety of applications, evaluation of controllers in the context of nonlinear applications will become much more common. Another reason for the lack of interest in the application of controllers to nonlinear systems with immeasurable output states is that in many, perhaps even most, cases it is easier and almost reasonable to redesign the system to allow for measurement of the output. In most geartrain systems this would be not only reasonable, but there is very little reason for the output to ever be immeasurable. In the case of a valve, however, the disconnect is resident in the actual body of the valve, between the input shaft and the valve itself. The internal connection makes it impossible to measure this state. This research is of particular interest, therefore, in the applications where nonlinear behavior manifests at the actual output of the system and not in an intermediate step, after which state measurements would be possible. However, once controllers have been evaluated and proven for such systems there is nothing to prevent their application to systems where they might previously have been avoided by adding measurement capabilities.

The different dynamic components of this project have been previously considered and evaluated. A number of different discussions of gear dynamics have been made over the years. Limiting the scope to standard, off-the-shelf gears it can be determined that there are standard formulas and assumptions which can be made about their dynamic response. The internal deformation and stresses within gears are critical to the understanding of the dynamic response of the gear itself. Analyses of the gear tooth under stress are also prevalent. The involute tooth profile has long been accepted as an industry standard, however, and its behavior is well documented.

primarily by AGMA, the American Gear Manufacturers' Association. The behavior associated with impact dynamics has been researched by a number of individuals. Dubowsky investigated impacts with respect to clearance connections (the phenomenon which creates backlash), creating numerous models and formulae which are still used as standard reference in this type of application. (Dubowsky, 1971) System dynamics for basic components like shafts are also industry standard knowledge. Motor characteristics are also easily found and most electric motor manufacturers will provide motor characteristics upon request which are based on experimental testing. These results are quite robust and are suitable for use in system design and modeling.

Controller design is a large and highly dynamic field. Advances are frequently arising which allow for new possibilities in the control of various systems. High level controllers are immensely complex and expensive to develop. In order to avoid such expense and complexity, established simple feedback controllers will be used in this project. Such controllers are considered industry standards. The use of feedforward control will also be explored in conjunction with standard feedback controllers. Feedforward control is well understood but not frequently encountered. The intended form of feedforward control is a set point shift where the set point for the system will be shifted in a direction dependent upon the rotational velocity of the drive system. This logical equation:

if(omega >= 0), then add a constant shift value to the set point
else, subtract the constant shift value to the set point

Equation 1.

can also be seen in the block diagram in Figure 3.

The systems have been developed which would allow for controller design and evaluation for applications which exhibit backlash. Those systems have not been combined and tested with the intent of evaluation of controller capability. The incorporation of nonlinear dynamic models into a simulation environment will allow for controller evaluation in the context of a limited state measurement system.

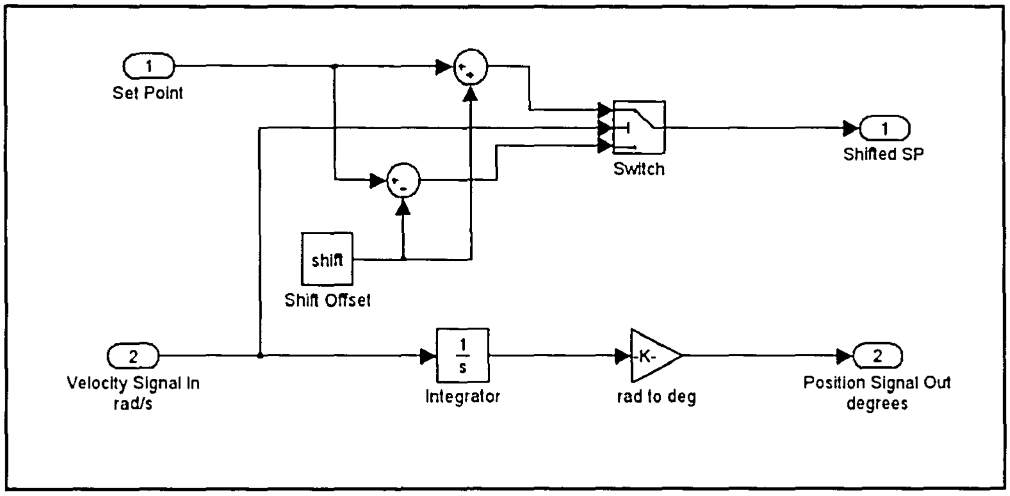


Figure 3: Set Point Shift, Feedforward Block Diagram

System Dynamics

The modeled system is a collection of individual components, each based on an accurate mathematical model of the physical equivalent. Each model was based on fundamental physical laws and standard theory. Mathematical models were derived from physical laws and Simulink blocks were developed to reflect these models. Each Simulink model was tested and shown to agree with the results predicted by mathematical theory and previous research.

The applied concept of deformation under load is central to an effective model of the behavior of a physical system. The dynamic response of a component must include the deformation it undergoes in order to account for the entirety of its behavior. For example, a rotating shaft will act as both a torsional spring and a rotational inertia (both types of energy storage). Without the inclusion of the rotational deformation, a system model could not account for the energy stored in the twisting of the shaft, but would only include the rotation of the mass itself. This type of energy storage can be critical to the behavior of a system. Especially important is the oscillatory motion that tends to occur due to the energy storage and release from a spring. It is also important to note that the process of deformation itself not only stores energy, but also absorbs a small amount of energy, leading to internal damping of dynamic systems.

Deformation in a geartrain is notable in two different components. A primary deformation exists in the shafts that support the gears and transmit the power through the system. The other notable deformation occurs in the gears themselves. These deformations can be visualized as a twisting in the different components when under torsion, as shown in Figure 4. The contact between teeth on facing gears causes deformation on the surface of each tooth, as shown in Figure 5. The initial contact is ideally modeled as a line contact, but the deformation of the teeth leads to a rectangular contact patch. This distortion is a considerable portion of the

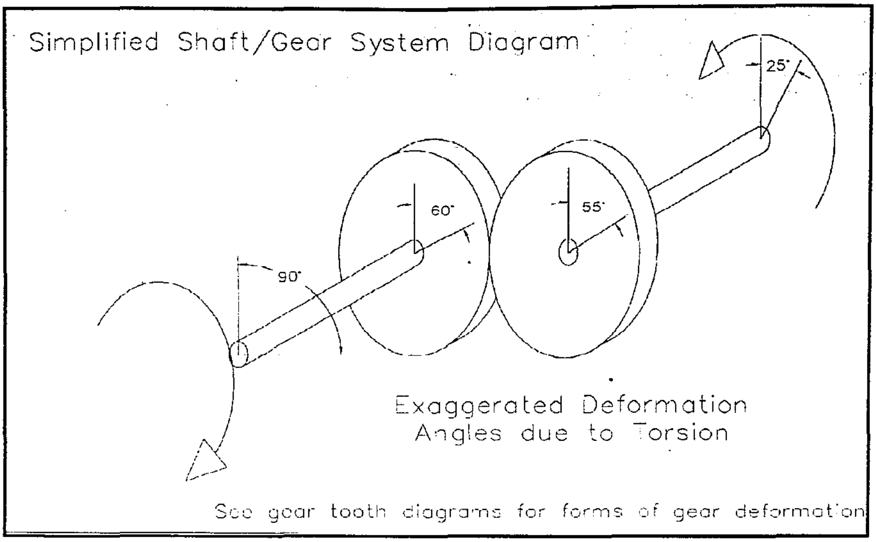


Figure 4: Diagram of Deformation in a Gear System

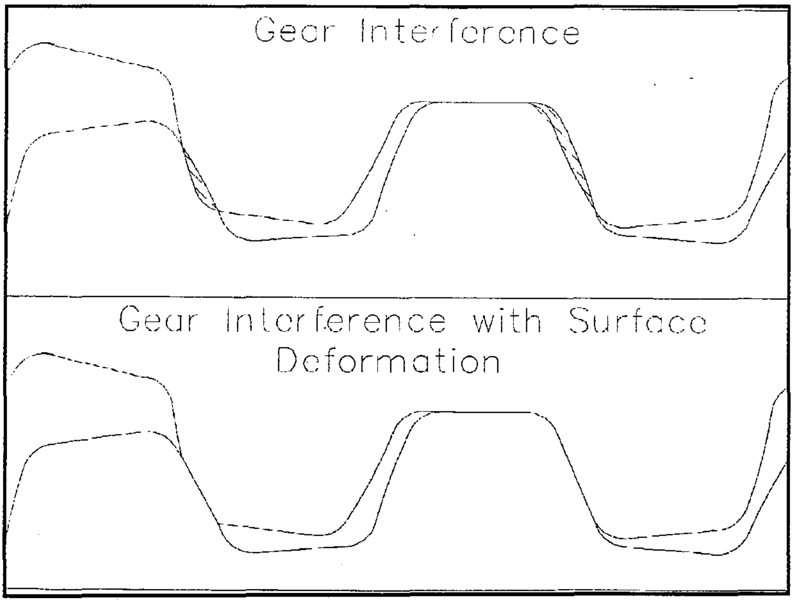


Figure 5: Gear Tooth Deformation at Contact Patch

overall deformation in the gear system. Additionally, each gear tooth acts as a single cantilever beam under load and can deform sufficiently to result in a large enough dynamic response that it cannot be reasonably ignored. That is, each tooth absorbs, in turn, a load applied across its radial axis as shown in Figure 6. Said load results in deformation of the tooth in the direction of the load, inevitably causing stress concentrations at the base of the tooth. The stress concentrations limit the load

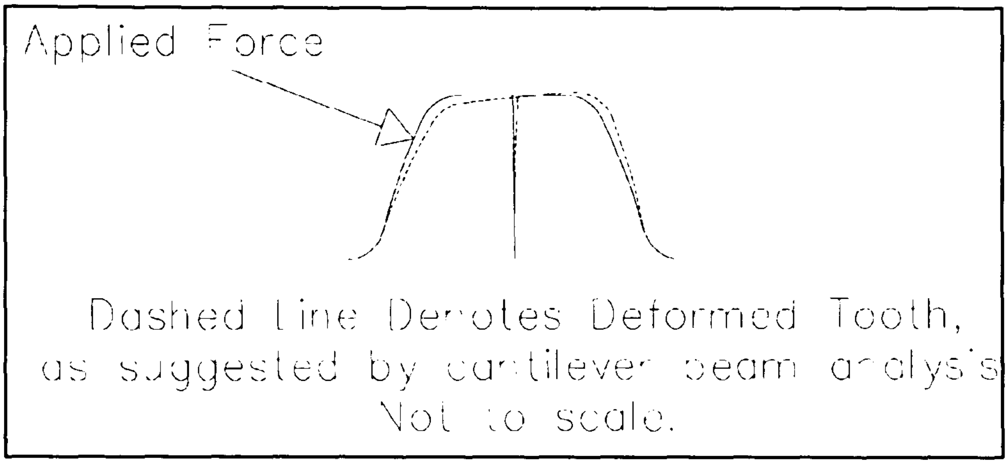


Figure 6: Gear Tooth Deformation as a Cantilever Beam

capacity of the gear, but well before the stresses become critical the tooth deformations affect the dynamics of the system. Deformation in the body of the gear does occur, but compared to the deformation of the teeth, a disc under torsion deflects minutely. Therefore the contribution from the gear body is significantly reduced in contribution to the dynamic response when compared to the contact and tooth deformations and is considered negligible.

The process of deforming bodies also absorbs energy. This is reflected as damping in the modeling of the system. Damping in the actual physical system is due to the material dynamics at a microscopic scale. While small enough to be ignored in most engineering solutions, its contribution is critical to dynamic models. Many engineering solutions are concerned with the failure of the component in question. In that case it is unimportant to include damping as it can only decrease the stresses internal to the component. In a dynamic response, however, even minimal damping prevents numerical solutions from failing due to the compounding of rounding errors and resonance responses. Damping is often present in systems due to the interactions between components. The internal damping in a body is often one to two orders of magnitude smaller than the damping due to outside interactions, but in high quality components it can contribute significantly to the dynamic response of the system. It is important to include the internal damping in the creation of system models, although once fully developed it may be determined that the internal contribution can be ignored, if its contribution is minimal, in the interest of a stable simulation or reduced runtime.

A primary component of this project was the accurate modeling of a non-linear system. This required utilizing common modeling components in the form of source, storage and dissipative elements. (Shearer, 1997) Assigning attributes and parameters in a system model is critical to the model results, as an incorrect parameter can drastically affect the outcome of any simulation. Understanding the different components and descriptions can make a large difference in the flexibility of a system model.

The most common components in most models are storage elements. Storage elements in physical systems are most often thought of as masses, although in rotational systems it is more accurately the rotational inertia of components which acts to store energy. These elements store physical energy in their motion, while others use a more active response to store energy, such as springs. Both types of storage components appear frequently in dynamic systems. There are also common storage elements in fluid, thermal, and electrical systems.

Source elements are also essential to most models, as otherwise the system would remain inert. Source elements are objects such as motors or actuators that interact with a system to actually drive a response. As with all elements, there are analogous elements in every type of system. One simplistic example is the battery in an electrical system which provides the "movement" of the circuit. It can be deceptive to refer to an item as a source, as in strict terms most sources are actually components which transfer energy from one system to another and are not actual sources. This is especially true with mechanical systems, where motors are often thought of as sources despite simply converting electrical energy to mechanical energy. Taking a different perspective, it can be seen that looking at any system, you can trace the energy flow through the system. At any point in the system, power flows from one component to the next. One side of that point can be seen as a source and the other side the load. Within a model, there is a starting point, and this system component is referred to as a source.

Dissipative elements, or damping elements, are the components that remove energy from a system. Without dissipative elements energy never leaves a system; the system conserves all of its energy. While in some limited cases this may be a reasonable approximation, in most cases dissipative elements make system models much more closely resemble physical systems. Common dissipative elements are used to model friction in systems. Known as dampers, many dissipative elements have no distinct component in the physical system which they model, but instead model practical and measurable effects of the interaction between and within components..

Finally, there are different elements which have non-linear responses which the common modeling elements cannot adequately describe. A common non-linearity which is the focus of this exploration is that of geartrain backlash. This non-linearity is used to describe the connection between two gears. This connective element accounts for the imperfections in all manufactured gears resulting in a slight disconnect between the two gears when the direction of rotation is changed. Without the non-linear modeling elements the system would behave as though there was a continuous connection between the gears. The logical equation for contact will actually find the overlap between the gears and requires the clearance dimension and the relative positions of the gears themselves and is shown in Equation 2. The block diagram for this process is shown in Figure 7.

if(|Xrelative| >= clearance/2), then ovelap = subtract the clearance/2 from the relative position
else the overlap = 0In many cases, this is not a necessary
Equation 2.

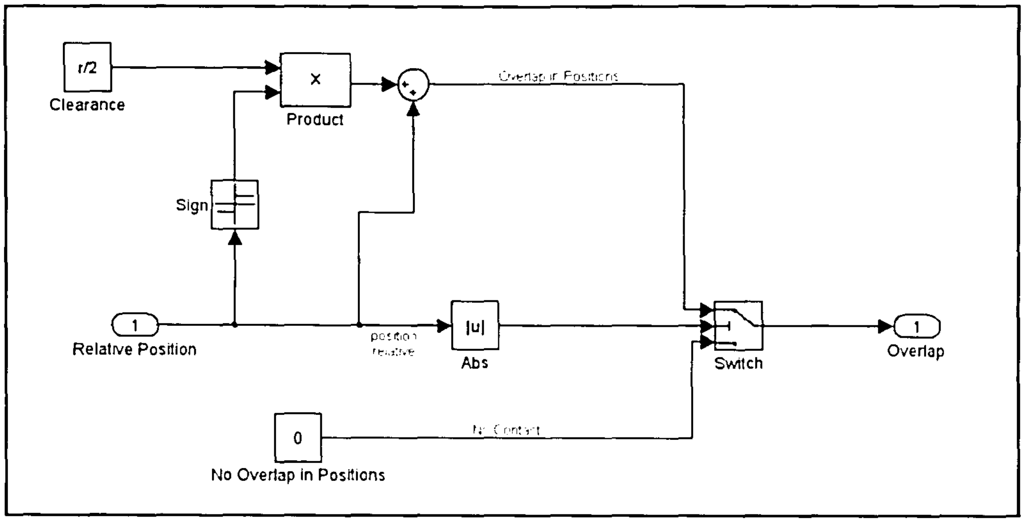


Figure 7: Backlash Block Diagram

inclusion in the modeling process, but in systems where high accuracy is necessary, backlash can present a necessary component of the system response.

The simplest component of the system is that of the mechanical shafts which connect the different components. The shafts effectively form relatively stiff rotational inertia, torsion spring and damper systems between the various components. Mathematical models are based on basic deformable body mechanics. The system is modeled as a pair of inertial bodies that are connected with a spring and are damped. In order to interface correctly with the other models, inputs to the system are considered to be the torque on one end of the shaft and the rotational velocity of the other end. With the two input states defined, the system can be solved and outputs can be used to interface with other system components.

The next simplest model is that of the DC motor driving the system. DC motors are well understood and there are a number of effective models. Different levels of model complexity are available; the simplest model available that includes the majority of factors in the dynamic response of the system is used in the simulations. This includes not only the basic electrical system response, but the inertial response of the mechanical system as well. The system equation was found to be represented by the following system of equations:

Rotational Speed = 1/Imech * int(Torque) where
Torque = F(system torque, electromagnetic torque, and rotational speed)
Electromagnetic torque = KT * i
i = 1/L * int(Voltage)
Voltage = Σ(Voltage in, R * i, rotational speed * Back EMF Constant)
Equation 3.

This system uses voltage and rotational velocity as inputs and solves for the generated torque and required electrical current as outputs as shown in Figure 8.

The component which is most critical and most complicated in the system is the gear pair itself. Including backlash in the geartrain model is done by including clearance between the meshed teeth of a gear pair and allowing impact between the teeth. The basic impact model is built from the work published by Dubowsky. (Dubowsky, 1971) The system is modeled as a pair of free bodies with clearances between them. As shown in Figure 9, as they bodies approach they are modeled to interact as though connected by a spring and a damper. However, when there is no contact, there is assumed to be no interaction between the two bodies. Dubowsky's

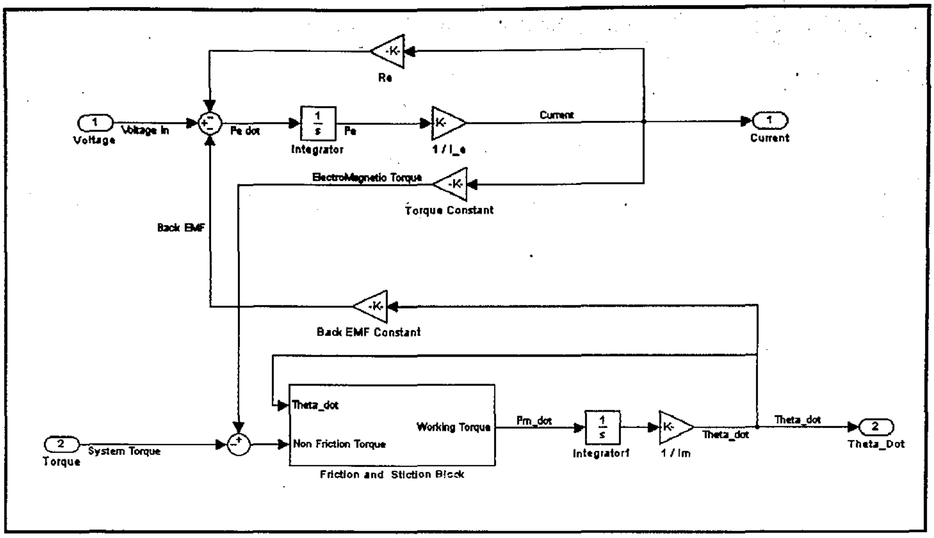


Figure 8: DC Motor Block Diagram

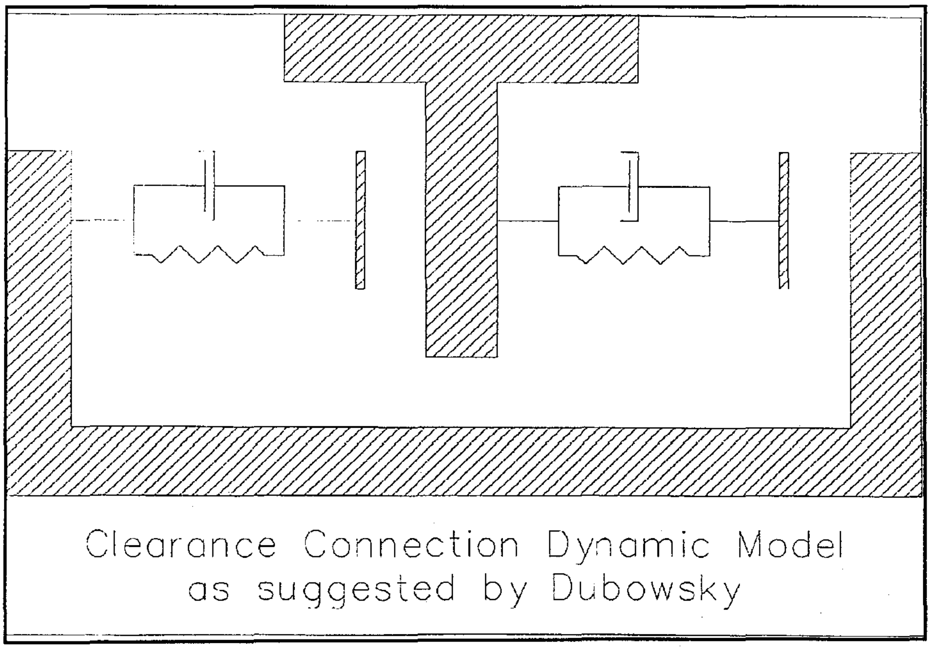


Figure 9: Clearance Connection Dynamic Model

solution to the impact pair is a purely mathematical model to the differential equation defined by the equations of motion. Using his model cast in a rotational reference frame, the system includes a clearance in the interaction between gears, duplicating the mechanics of backlash. This manifests as a discontinuity or nonlinearity when the driving gear changes direction. The impact causes a rebound in addition to the transfer of energy from one gear to the other, allowing for multiple impacts. The process of multiple impacts due to a single input is known as chatter. There is also significant damping in the system. The damping in a geartrain has many sources, including the sliding of one tooth on another, the loss of energy in an impact due to the deformation of a material, and imperfections in the gear surfaces. The model suggested by Dubowsky was chosen as a primary model. (Dubowsky, 1971) The model proposed by Sarkar was also considered, as was that investigated by Shing in his research. (Sarkar, 1997) (Shing) The impact model proposed by Dubowsky is the simplest and yet appears to fully describe the dominant behavior of the system. It should be noted that Sarkar's explanation of the stiffness and damping are more clearly defined as to the physical sources and clear relations to basic material properties.

Controllers are the intelligent part of an active dynamic system. The controller exists in order to cause a system to behave in a desired manner. Basic controllers are based on the addition of a corrective command that is created to shift a parameter towards a set point. The simplest controller is an on/off, or bang bang, controller similar to those used in a thermostat. In that specific application, when the temperature in a room exceeds the limits imposed by the operator, heat or air conditioning is turned on until the room temperature is back within the set range. More complex controllers vary the response in relation to the magnitude of the variation outside the set range. When faced with a nonlinear system, a standard controller does not consider the non-linear response when determining the proper magnitude of the corrective response. This is the concern in using a standard controller to manage a system with distinct nonlinearities. This will not prevent a standard controller from managing a system, but it will reduce its effectiveness, both in response time and error deviation.

There are a number of different basic controller methods that were tested in order to accommodate a system that exhibits backlash. Bang/bang, proportional (P), and proportional/integral (PI) were implemented in the test system (Figures 10-12). The system equations for the controllers follow:

Bang/Bang Control, requires a range, band
if(error > band/2) then multiply the sign of the error by the maximum signal strength
else output zero signal

Equation 4.

Proportional Control, requires a gain, KP
Output = error * KP

Equation 5.

Proportional/Integral Control, requires gains KP and KI
Output = KP* (error + KI * int(error))

Equation 6.

The Proportional and PI controllers feature an additional block in the diagrams, a saturation block, which prevents the controllers from driving the system beyond its operating parameters. This feature does not affect the calculations by the controllers except to limit the magnitude of the output signal. A large range of parameters on a

PI controller should be able to account for most systems, although a proportional/integral/derivative (PID) controller can be more robust, if properly matched to the system. Due to the nature of a backlash non-linearity it also appeared that a set point shift should be tested which would shift the internal set point for the system whenever the drive direction is changed. This shift should account for the non-linear behavior of the backlash itself. This type of response is known as a feedforward control loop. It is used to enforce a known deviation on a system as soon as it can be predicted. Used in conjunction with a standard control scheme, this could yield an effective, efficient and simple control for the non-linear system.

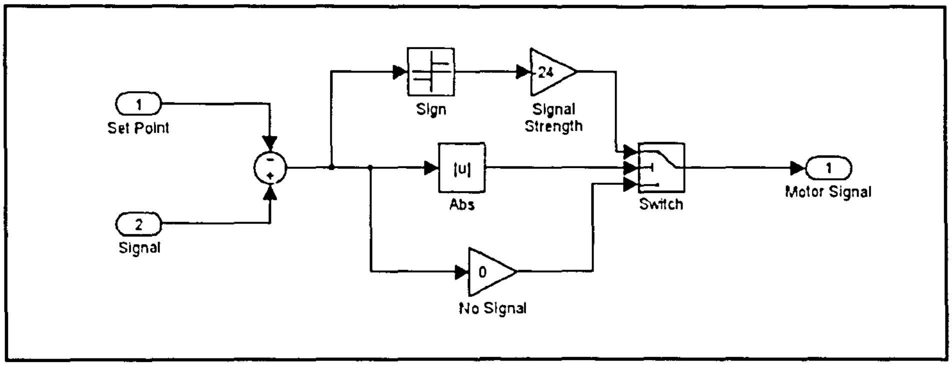


Figure 10: Bang Bang Controller Block Diagram

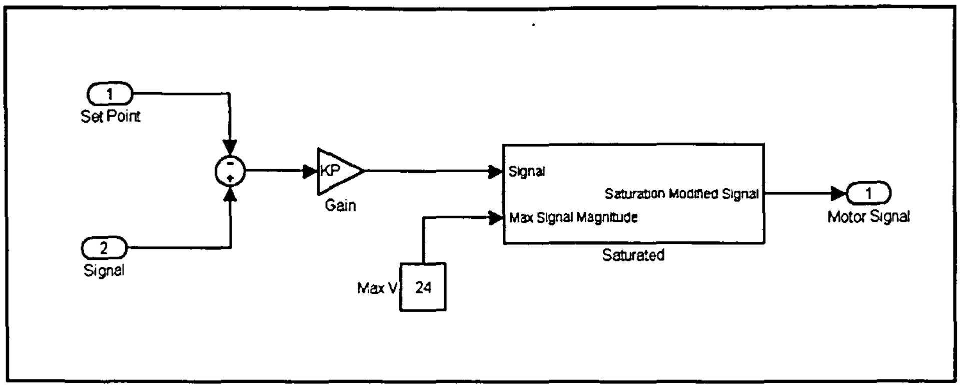


Figure 11: Proportional Controller Block Diagram

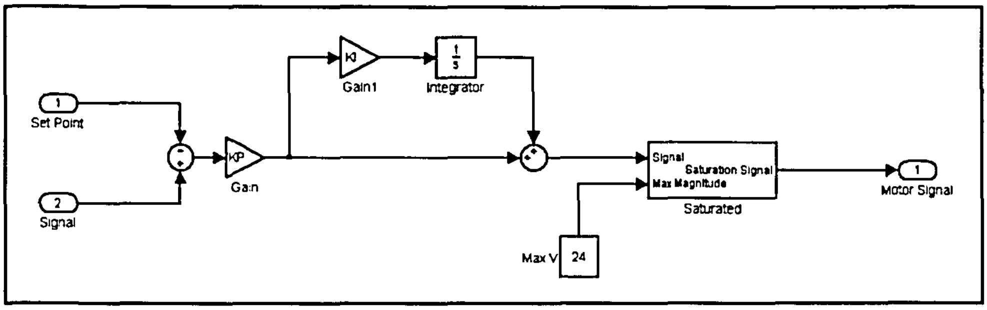


Figure 12: Proportional/Integral Controller Block Diagram

System Model

The Simulink models were built to replicate the behavior of the mathematical models of the physical geartrain system. In order to structure the models in a coherent manner, the mathematical models were restructured using bond graphs. bond graphs clearly define the relationship between the different elements in each model and were used to form the mathematical equations that were used to form the Simulink models. (Samantaray, 2001) The bond graphs with causality strokes imply the mathematical models and can be converted directly to block diagrams. The desired effort or flow can be followed back through the bond graph and the relationship between the desired element and all of the other components of the bond graph are found. The bond graph for a simple shaft is shown in Figure 13. In Figure 14 the bond graph is shown with suggested alterations to avoid the derivative causality implied on the source side inertia in the bond graph. After applying this process to each component, such as the gear pair in Figure 15 and the DC motor in Figure 16, the different blocks were assembled into a larger block diagram in a manner which resembles a ladder, with each block forming a rung. The connections between the inputs and outputs of each block form the sides of the ladder. This is a graphical indication of the dependence of each component on the behavior of the others and is shown in Figure 17.

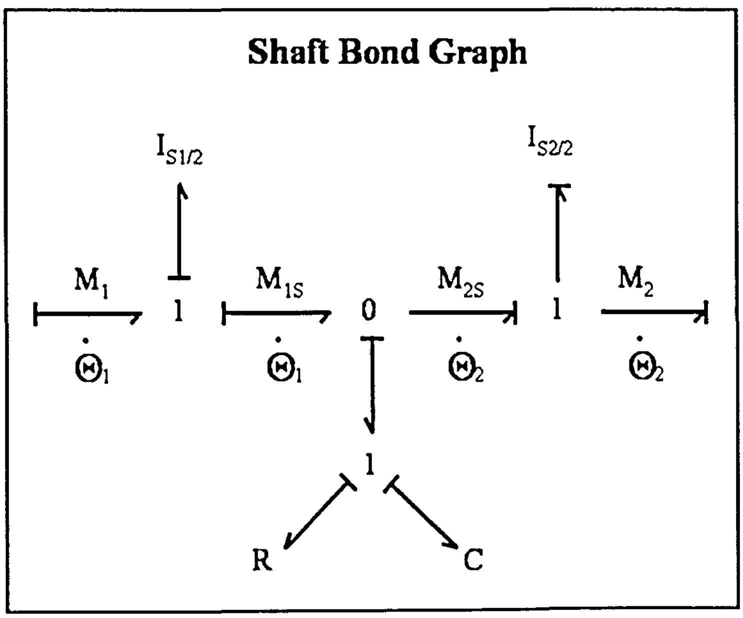


Figure 13: Shaft Bond Graph

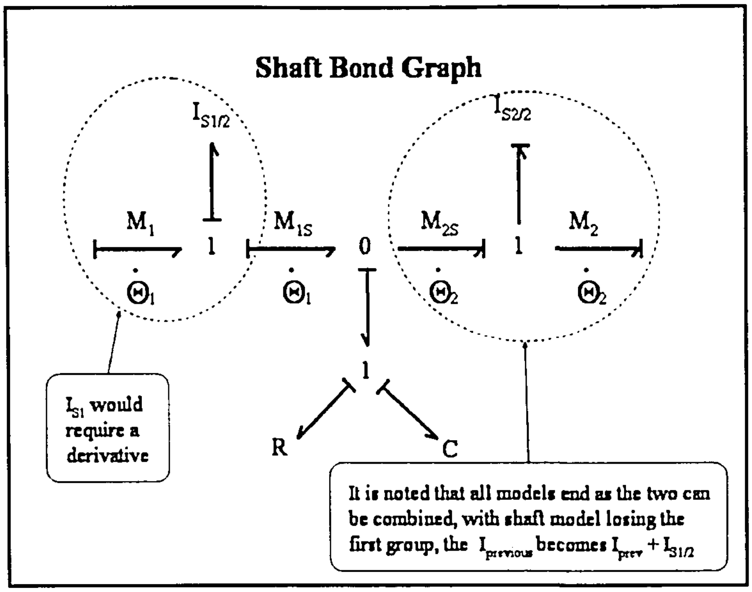


Figure 14: Shaft Bond Graph, Removal of Derivatives

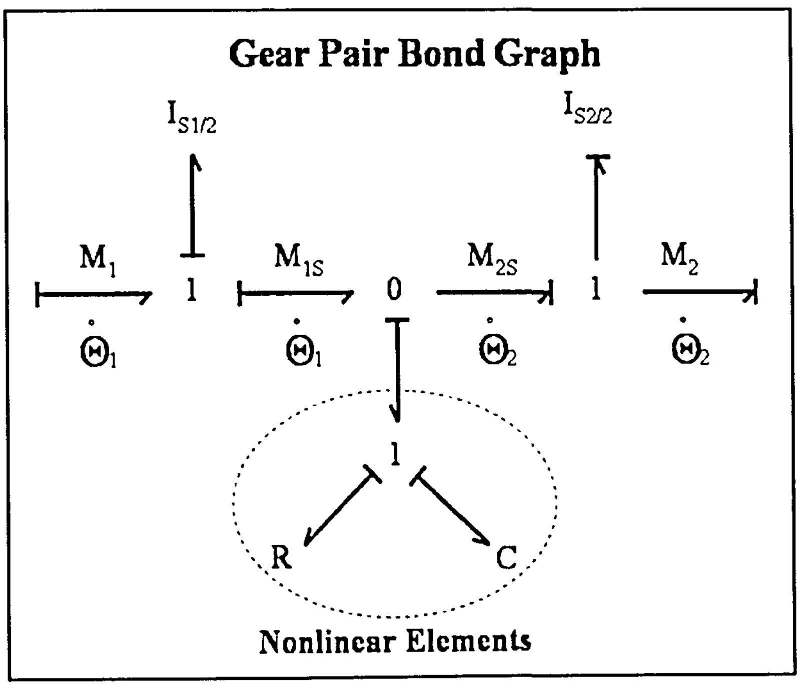


Figure 15: Gear Pair Bond Graph

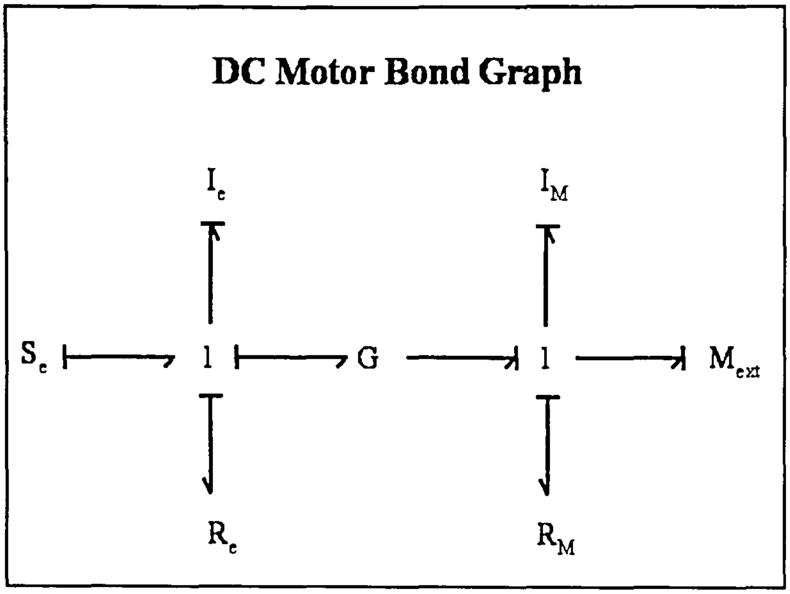


Figure 16: DC Motor Bond Graph

Basic models were formed using equations of motion and knowledge of the basic behavior of physical systems. Most of the components were modeled as a pair of masses or inertias with the connection between them defining the behavior of each individual system. Models of each system component were derived and tested independently. After confirmation that each component functioned as expected and could be solved under a variety of input conditions the models were combined and testing of the overall system was begun. The inclusion of derivatives in the system models was found to cause immense error propagation due to rounding practices and software limitations. This caused the models to fail when run in pairs or combinations of more than two. The lone exception for this was when a shaft model preceded a gearbox model. It would seem that the nonlinear behavior of the gearbox isolated the second derivative in the system for enough of the run time that the system did not fail. The system dynamics were reevaluated and solutions were sought which did not require the inclusion of derivatives in the model. Using Bond graphs a system of equations was found which would possibly allow for a set of equations that did not rely upon the implementation of derivatives in their solutions. Each model at first appeared to require a derivative component in its solution, but upon further inspection it was found to be possible to combine those aspects of each model with other models, using basic lumped models, thus avoiding the derivative requirements.

The importance of avoiding derivatives is unique to the use of numerical solutions. Integrals, in numerical solutions, are much more forgiving of rounding errors and requirements as well as high frequency responses and frequent variations in input. In a feedback system that utilizes derivatives, a high frequency response will cause an effective positive feedback loop which is highly unstable. An integral applied to the same system will absorb the variations and effectively dampen the speed of and average the magnitude of the response. This tendency is a marked departure from an analytical solution, where integrals and derivatives have the same mathematical validity. The difference between analytical and numerical models must be recognized if a valid solution is to be expected.

In general terms, the systems which are modeled individually are a motor, a shaft, a gearbox, and a general load. Each system has inputs and outputs that are unique to its solution and are used to interface with the other modeled components, as shown in Figure 17. With the exception of the load, each model has both a pair of inputs and a pair of outputs. The load model has a single input (omega) and a single output (torque). Each mechanical model uses a torque and a rotational velocity (omega) as inputs and a corresponding torque and omega pair as outputs. In the models that were created for this investigation, each system had one input from each modeled end, and one output. This forces the models to be used interdependently; each model is dependent upon the solution of those connected to it for its own solution.

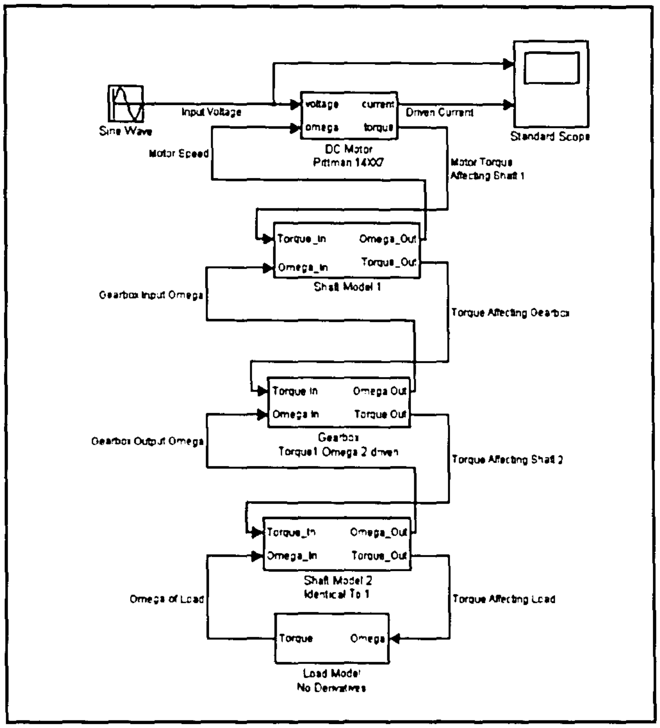


Figure 17: Block Diagram in Ladder Organizational Layout

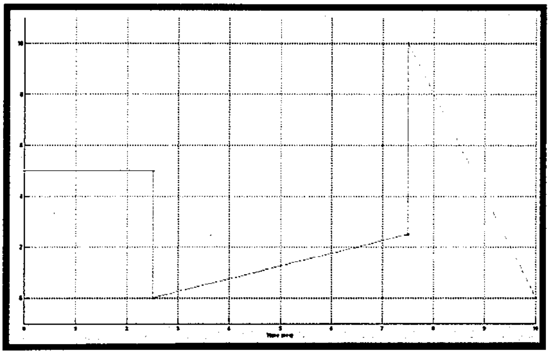
The component of most interest in the system is the gearbox. The system is modeled as a pair of masses which have a clearance between them. Their motion is independent until the clearance has been removed by the motion of one or both of the bodies. At this point, the interaction between the bodies is effectively governed by a spring and damper system. The accumulated momentum in the bodies tends to cause the spring to store enough energy to cause the gears to separate after very little interaction, however. This process very closely replicates the dynamics of an impact. The response has been compared to experimentally gathered data of gear impacts and the responses correlate well. (Dubowsky, 1971) While theoretically a simple system, the dynamic response of the nonlinear system model is rarely simple. The system often exhibits chatter and causes extremely nonlinear and undesirable motion in the entire system. This of course is the reason for interest in this system model and the various responses to the system under conventional and experimental control methods.

The model development process for the Simulink representations of the physical models was a critical element in the transition from theory to numerical simulations. The basic mathematical implementation was made directly from previously derived equations and block diagrams. As the model was developed, each internal component was tested as it was developed to ensure that its response reflected the design intent. Known inputs were connected to the system blocks and the output was monitored, demonstrating static and transient responses. If problems were discovered, the block was inspected for obvious faults. If no faults were immediately discovered, additional signals within the block were monitored and the system was run again. Different components were scrutinized until their response matched the expected response. Difficulties were more evident in blocks where the Simulink solver could not complete a simulation. In these cases a primary suspects were feedback loops. One method for determining if a feedback loop was causing instability was to add a gain of zero to the loop, thus disconnecting it or removing it from the system. If the system became stable, it was determined that this loop was causing the failure and required further inspection; if this did not alter the results of an attempted simulation, the loop was restored and other loops and components were inspected and tested. Another key feature that caused failures was the use of inaccurate constants. There is a critical requirement for reasonable constants to be used in the description of system components. If the inertia, stiffness, or damping of a system were unreasonable, they quickly caused feedback loops to become unstable, despite the experimental knowledge to the contrary. This is a particular danger when being provided component specifications in English units, as notation can be unclear as to whether a measurement was described as including a force or a mass.

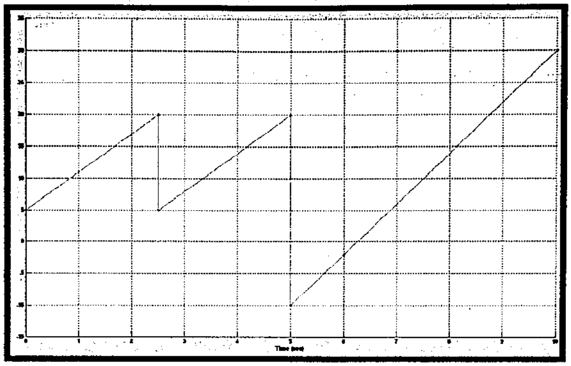
Simulations

The simulations tested were created with the intent of creating situations which excite the nonlinear behavior of the geartrain to be clearly manifested in the response of the system. Three different desired output traces were created in order to show responses under different types of dynamic demands. The output of the system was compared to the desired response and the deviation of the system from the desired output was quantified. The different controllers could thus be compared. It is possible that different controllers would respond better for different dynamic demands. The basic controllers tested are a bang bang controller, a proportional controller, a proportional and integral controller, and a proportional, integral and derivative controller. Each controller was tested with and without a set point shift mechanism, a feedforward control, which takes some of the kinematics of the backlash into account.

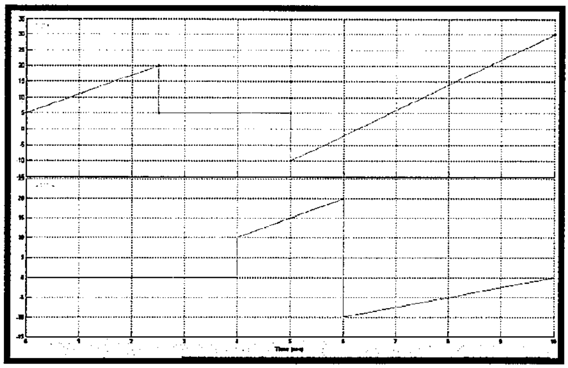
The simulations were limited to ten seconds of simulated time, preventing the data set from becoming unreasonably large. The limited runtime is long enough to present the limitations of the various controllers, however, and allows for reasonable comparisons to be made between the controllers. One analysis will be an integral of the error of the output compared to the desired output. This gives weight to both the magnitude of the error and the length of time which the system remains in error. The set point profiles are shown below.



Profile #1



Profile #2



Profile #3 – Summation of the Pair of Profiles Shown

Figure 18: Simulation Profiles

The first profile includes an initial dwell, two step changes, and two uniformly sloped sections. The second profile has two step changes and three uniformly sloped sections. The third profile includes a dwell in the middle of the profile, a section where the slope changes, and four step changes. Each profile progressively requires larger deviations from zero, requiring higher input voltages for longer periods of time. Step changes are easier for the bang bang controller to respond to, while curves are more difficult for the simplest controller. The proportional and PI controllers do not respond quite as quickly to step changes, but are better at following curves closely. All controllers can be made to perform better in situations by changing their operating characteristics. A bang bang controller has a range in which it does not output a signal and by reducing this range it will more effectively follow a profile. However, there are limitations in signal sampling and sensor resolution which can limit this technique for improving bang bang controllers. These controllers are also more likely to be affected by wear as they must be constantly switched on and off as needed to follow a profile; as often as every 0.0002 seconds in simulations. This type of resolution may also be unrealistic, giving higher resolution to the bang bang controller than it would actually be able to achieve. Proportional and PI controllers are more expensive than bang bang controllers and require more skill to implement properly. Once properly implemented in a system, however, they offer much more precise control and can be changed more easily if the system changes around them. Bang bang controllers cannot be altered to reflect system changes and will be much more effective on some systems than on others, depending on the dynamic behavior of the system. Bang bang controllers are most effective in systems which are naturally overdamped and require high effort to affect the behavior.

Simulations were made using a modified Rosenbrock solver, internally identified in Simulink as ODE23s. This solver is described as stiff, indicating that it is robust and effective applied to systems which oscillate at high frequency. It was chosen over other solvers by trial and error. While time measurements were not taken, it was readily apparent that this method reduced simulation runtime considerably when applied to the overall system. Other solver methods, such as the ODE45 method, which is a Runge-Kutta numerical method, while stable, required significantly more processor time to complete the same simulation. In order to confirm that the ODE23s solver provides sufficient accuracy, a few simulations were completed with the ODE45 solver to provide comparative results.

Results

The primary goal of this project was to produce Simulink elements which would provide an accurate simulation of a reasonable physical system. The system that was created contains a model of a motor and a gearbox, with an attached load. Models for shafts were also created, but after analysis of the dynamic response of the system, it was determined that they would not substantially alter the system output. The shafts were incorporated into the motor and gearbox models, providing the proper inertial resistance to the system, but without adding increased complexity, and thus runtime, to the simulations.

The exclusion of the shaft models was undertaken after an analysis was made of the natural frequency response of the shaft compared to the gear mesh behavior. The gear mesh's natural frequency is approximately 110 kHz, compared to a natural frequency of about 600 Hz for the shaft. As there is a difference of approximately 3 orders of magnitude between the response frequencies, it was determined that the gearbox response would clearly dominate and that it would be an acceptable approximation to remove the stiffness and damping responses of the shafts from the system model. The shaft models were not deleted and could still be used in later models. If the gear mesh was less stiff, in fact, it would be necessary to include the shafts as their response to reasonably affect the response of the entire system.

The geartrain response appears to be reasonable, reacting as expected under various input signals. When subjected to the same conditions as Dubowsky used in his derivations of the impact pair model, his results are duplicated without difficulty. This is a key demonstration of the proper response of the system. The geartrain also exhibits a large amount of 'chatter' which is consistent with the combination of such a high frequency response and a clearance between components.

When controllers are applied to the system the simulations were able to provide useful information which could be used to compare controllers in a reasonable approximation of a work cycle. The results from a set of simulations the three tested set point profiles are compiled in Table 1. This data set was obtained with one of two K_P values (50 or 200), a K_I of 0.005, and a zero output range for the bang bang controller of 0.25 degrees. It was determined by Hurwitz criteria that the range for stability of the system under proportional control was that K_P be greater than 1. Analyzing the system response with gains of 0.5, 5, 50, 500, and 5000, it was determined that the optimum gain for the system would be achieved between 50 and 500, where the controller would not constantly feed a saturated signal into the system, but would still have a fast response time. Additional simulations were executed with a K_P of 200 in order to determine the changes which a different proportional gain would provide compared to the gain of 50.

The second simulation to be modeled was of an experimental system being constructed as an undergraduate project. This required the determination of the system constants for the model and altering the output from a rotational system to a translational system. After these changes were made, the system was simulated to demonstrate an expected response to an input signal. Profiles were made to demonstrate controller abilities. The system and simulation model can be compared after the construction is complete. This will be a primary instructional utilization of a simulation based on this modeling. The system model was simulated given a number of standard input signals in order to demonstrate the output behavior of the system.

Profile	Controller	Error, Integrated	Error, Integrated, Set Point shifted
1	Bang Bang	13.614	1.880
1	Proportional, K_P=50	14.160	1.959
1	Proportional/Integral, K_P=50	14.025	1.960
1	Proportional, K_P=200	14.219	1.858
1	Proportional/Integral, K_P=200	14.181	1.857
2	Bang Bang	22.925	3.561
2	Proportional, K_P=50	22.582	3.691
2	Proportional/Integral, K_P=50	22.567	3.684
2	Proportional, K_P=200	22.561	3.697
2	Proportional/Integral, K_P=200	22.587	3.670
3	Bang Bang	36.870	24.166
3	Proportional, K_P=50	36.779	23.791
3	Proportional/Integral, K_P=50	23.949	24.065
3	Proportional, K_P=200	36.644	22.926
3	Proportional/Integral, K_P=200	36.784	23.311

Table 1: Simulation Results

As a primary teaching tool, the combination of a numerical simulation and a physical system allow for unique opportunities. Basic experimental data can be gathered and compared to calculated data from the simulation of the analogous system. This would allow for verification of the simulation as a reasonable approximation of the physical system. The initial simulations and experimental data collections should be in response to a step change in the voltage applied to the system. This would provide data on the transient response of both the physical system and the simulated model and allow for the alteration of the simulated model to properly reflect the physical system. Later testing could be implemented to attempt profile matching on both systems. Accurate data collection for the experimental procedure includes the ability to make high frequency measurements of the translational output during the testing procedure. Without this measurement the systems cannot be properly compared and worthwhile comparisons are more difficult to obtain.

Discussion

The modeling approach required the creation of multiple component models in a manner which would enable multiple combinations of components. In practice this became less necessary than anticipated due to the dominant response of the gearbox. Once the system dynamics of each component combination were known, the shaft models were found to be more complex than needed for reasonable simulation response and the model was simplified to reduce the overall complexity. The shaft inertias were still considered important to the overall system response and were incorporated into the models of other components. This step could have been avoided if the full theoretical dynamic models of the system were investigated before the simulation models were created. While not particularly difficult or complicated, the system models were, in the final result, unnecessary. Exploration of the theory behind the system is important and fully understanding the theory is requisite. Mathematical testing that demonstrates components are not needed in final modeling can render some models obsolete, possibly before they are fully developed.

The system response to the inclusion of derivative functions in the system models is a tendency towards failure. This response is due to the numerical approximation of derivatives and the very high frequency responses in the system. However, with careful analysis of each system model and its corresponding bond graph, it was determined that with the inclusion of some inertias in connected blocks all of the derivative functions could be avoided throughout the system model. This necessity is a shortcoming of numerical methods in the solution of systems of differential equations and not specific to those methods used in Simulink. Once the limitations of the methods are understood and the system designed with these weaknesses in mind, a robust model was developed and implemented.

The modeling approach has certain weaknesses which must be acknowledged. A primary weakness that exists in this simulation method is the propagation of errors in the signal through the system solution and the potential for increases in the error and instability due to that propagation. In defense of the method used, essentially every numerical solution system shares this weakness to some extent. This solution method does not follow the same format as many published and commonly practiced solution methods. Thus the system may not appear to necessarily agree with initial expectations of the model of the system. However, the system does satisfy the full complement of equations of motion and has been verified to be an accurate representation of the physical system. Finally, the modular construction of the system model meets the design requirement of being easily constructed and modified. However, the system is difficult to change more than superficially, making changes to the monitored, output data difficult as not all signals are accessible without considerable changes to many levels of the model system. Thus the output from one experimental inquiry may not provide all of the information necessary to make alternate evaluations; substantial changes to the component models and the external model are necessary to change the output signals for monitoring.

The models themselves are quite robust, functioning properly even when subjected to unrealistic system demands. The simulations appear to require a reasonable response, never moving in a manner which appears unrealistic or even as though driven by mathematical rounding. The models' responses stabilize quickly under most conditions. The only sensitive aspect to the models themselves is a necessity for properly matching system parameters. If the system description would not be viable in a physical model, the simulation will not function either, usually failing while trying to solve the set of equations. Improper parameters are the primary cause of simulation failures, particularly with proven models. Indeed, during testing it was determined on numerous occasions that failures were directly due to incorrectly assigned constant values.

The model simulations under the influence of various controllers demonstrate the usefulness of the simulation model, the system behavior with various control schemes, and the relative strengths of different controllers. The model itself provides a testing environment for different controllers. It is unimportant if the system does not meet requirements or if it fails, as would be the case in a physical system. This is a major attraction of simulated system models, being able to test various configurations without regard for failure. This allows for testing of more aggressive schemes than would be possible on a physical system. Schemes which are not known whether they would provide a suitable response in a particular environment are also tested without concern of failure in a simulated trial. The simulations, under the influence of the controllers, were remarkably similar. The different controller schemes only yielded minimal changes in overall error or output profile. The feedfoward controller did markedly reduce the error compared to the simulations which were made without it, however. The different controllers exhibit different strengths after analysis of the simulations. The bang bang controller is the most effective when responding to larger and faster changes in the set point. This is likely due to the necessity of the controller to operate at maximum voltage or no voltage; this would cause the fastest response available. The weaknesses of the bang bang controller are twofold. The first weakness is the dependence upon the size of the hysteresis range wherein the controller gives no input to the system. The second weakness, which is not modeled in the simulations, is a limitation on the sensing and switching resolution for a bang bang controller; a realistic controller would not be able to reevaluate and respond to its inputs every 0.0002 seconds. The proportional control does not exhibit this latter difficulty, but the hysteresis error manifests itself in the reduced response to smaller errors. This can be overcome by increasing the gain value, so proper tuning is critical to satisfactory response. The proportional controller does exhibit some steady state error, which is dependent upon the system characteristics and would likely be minimal in the simulated systems, but the deficiency is notable. Proportional/integral control retains the strengths of the proportional controller, but the integral component to the controller will eliminate the steady state error. As with proportional gain, the integral gain affects the response time of the integral component of the control and can be increased for faster response.

The additional testing of a higher proportional gain value of 200 demonstrates both a strength and a weakness of PID family controllers. The overall error for the simulations with the increased gain varied dependent upon the profile. In some cases it appears that the gain was large enough to cause overshoot of the intended position, and the resulting oscillations (particularly with a set point shift) tended to increase the output error. However, where the proportional and PI controllers were less effective than the bang bang controller when used with a gain of 50, it is apparent that increasing the gain made them much more effective, reducing the error significantly over the course of a simulation. This discrepancy between improved and decreased performance based on intended use of the controller reinforces the requirement for proper testing and tuning of the controller.

The results of basic testing demonstrate that the Proportional and PI controllers exhibit improved responses to various profiles over the bang bang controller. Logical extension of simulation results and basic controller knowledge indicates that for systems with constantly shifting set points a Proportional controller would be sufficient and would provide reasonable response with proper tuning for the intended application. However, with longer periods of dwell at constant values, the PI controller would reduce the incidence of steady state error and improve the overall system response. The necessity for tuning the integral loop of the PI controller is seen in the increased error in the trials. The integral portion causes increased overshoot compared to the proportional response. Either the proportional gain would need to be raised or the integral gain lowered in order to compensate for this tendency. The bang bang controller has the ability to provide adequate control over many systems, particularly highly overdamped systems which require large magnitude, fast changes in their responses. However, the bang bang controller has many physical limitations which must be more closely examined before integration into a system.

The feedforward addition to the system, referred to as a set point shift, does reduce the overall error, as expected. However, it does cause the Proportional and PI controllers to behave similar to the bang bang controller. This occurs due to the fact that whenever the set point is overshot and the controller attempts to compensate for the error there is an additional, instantaneous error associated with the set point shift. This will quickly cause the controller to produce a large input in order to move the system towards the new set point. This process is repeated many times over. It actually replicates and possibly exacerbates the natural problem of chatter in the gear mesh. The advantage that the Proportional and PI controllers have despite this is that they will reduce the input signal as the error approaches zero, reducing the impact velocity and thus some of the overshoot. This behavior is also very dependent upon the system dynamics. If the system is heavily damped, the overshoot may not occur, allowing the system to settle and reach a steady state condition. This is also an advantage of the P and PI controllers, as the bang bang controller has difficulty reaching a state where it can remain off due to overshoot problems.

The translational system exhibits the effectiveness of the simulation model as an analysis tool. Using the model it was determined that the total speed the system was capable of was about 0.176 inches per second. This limit is primarily dependent upon the friction and other resistive elements within the gear system; reducing these elements would increase the response speed of the system output. An option for increasing the response speed of the system would be to utilize a stronger motor, this is being considered by the undergraduate team.

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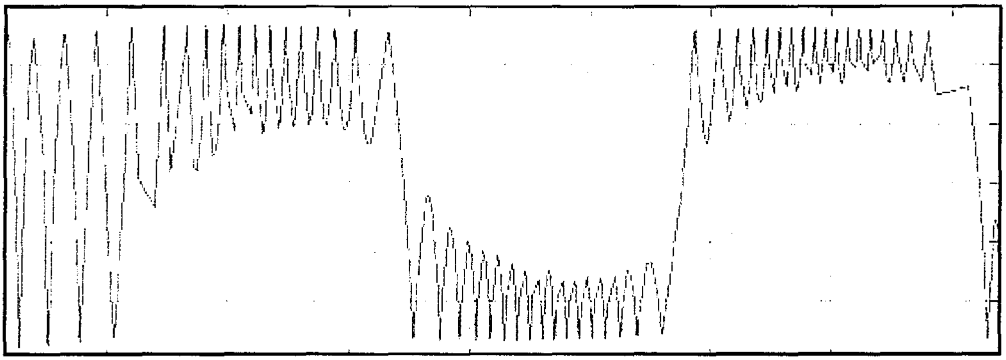
Appendix A

The relative position graph for a gearbox simulation where the input position is forced to follow a sine wave. The output gear of the gear pair is given no input and its only interaction is with the input gear. This response is predicted by Dubowsky for an impact pair and verifies the gearbox model to be an accurate rotational impact pair model.



System Response under Dubowsky Criterion

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